Wall Roughness Modelling with k-w STT Model

B. Aupoix

WALL ROUGHNESS MODELLING WITH $k-\omega$ SST MODEL

B. Aupoix[1]

[1] ***ONERA – The French Aerospace Lab, F-31055, Toulouse, France***

bertrand.aupoix@onera.fr

1 Introduction

RANS turbulence models are developed for smooth surfaces but, for many applications such as flows over aged surfaces, e.g. compressor or turbine blades, or iced surfaces, wall roughness can play a significant rôle.

Only k-type, distributed roughness are considered here. A key feature of the flow over rough surface is that, as pointed out by Nikuradse (1933), the logarithmic law is preserved but shifted, as shown in figure 1, where $^+$ denotes wall scaling, i.e. variables made dimensionless with the friction velocity and the viscosity. As the outer region is preserved, it can be shown that the drag increase is directly linked to the velocity shift Δu^+. Various correlations were proposed to model this velocity shift w.r.t. the equivalent sand grain roughness k_s^+ and are presented in figure 2, pointing out significant differences in the transition regime for $k_s^+ < 30$ as well as a higher level for Nikuradse's correlation in the fully rough regime ($k_s^+ > 100$).

Various approaches can be used to compute flows over rough surfaces but the equivalent sand grain approach, which is more empirical and only alters the turbulence model to reproduce the drag increase due to the wall roughness, is the only affordable tool for industrial applications.

The $k-\omega$ SST model is the workhorse of the aeronautical industry. Wilcox (1988) proposed a wall roughness correction together with his $k-\omega$ model, but it was shown that this correction interferes with the SST limiter. Cures to better reproduce the wall roughness effects with the SST model were proposed by Hellsten and Laine (1998) and by Knopp et al. (2009).

The aims of this paper are to analyse the behaviours of the various wall roughness corrections, to evidence their drawbacks, to develop new corrections and to validate them.

2 Roughness model analysis

Simplified analysis

As most $k-\omega$ models have no wall damping functions, roughness corrections which aim to enhance turbulence in the wall region can only be based upon modifications of the turbulent kinetic energy and specific dissipation values at the wall. In order to investigate the rôle of the existing corrections, a simple analysis is proposed. It is based upon the fact that, unphysically, RANS turbulence models provide a unique solution in the wall region, once written in wall variables.

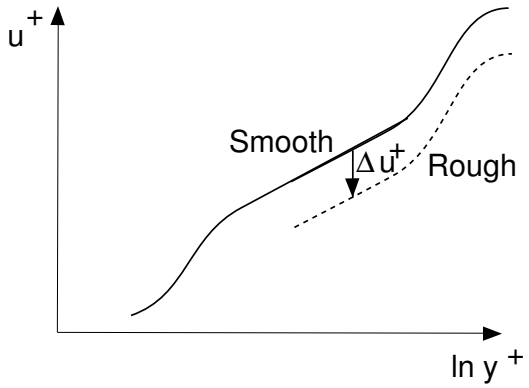

Figure 1: Velocity profiles over smooth and rough walls plotted in wall variables

Therefore, the flow equations in the wall region are solved, neglecting pressure gradient. For incompressible flows, neglecting advection, the momentum and $k-\omega$ model transport equations reduce to

$$
\begin{aligned}
1 &= \frac{\partial u^+}{\partial y^+} - \langle u'v'\rangle^+ = \left(1+\nu_t^+\right)\frac{\partial u^+}{\partial y^+} \qquad \nu_t^+ = \frac{k^+}{\omega^+} \\
0 &= -\langle u'v'\rangle^+\frac{\partial u^+}{\partial y^+} - \beta^* k^+\omega^+ + \frac{\partial}{\partial y^+}\left[\left(1+\sigma_k\nu_t^+\right)\frac{\partial k^+}{\partial y^+}\right] \\
0 &= -\gamma\langle u'v'\rangle^+\frac{\partial u^+}{\partial y^+}\frac{\omega^+}{k^+} - \beta{\omega^+}^2 + \frac{\partial}{\partial y^+}\left[\left(1+\sigma_\omega\nu_t^+\right)\frac{\partial \omega^+}{\partial y^+}\right]
\end{aligned}
$$

Equations are solved by a time marching procedure, on a very fine grid, grid convergence being checked. Wilcox' $k-\omega$ or Menter SST inner region model can be investigated, just by changing the values of the constants σ_k and γ.

For a smooth wall, k^+ is null at the wall while $\omega^+ = \frac{6}{\beta {y^+}^2}$ is imposed at the point above the wall. For a rough wall, finite values of k^+ and ω^+ are imposed, according to the considered roughness correction. The boundary conditions in the logarithmic region are the equilibrium logarithmic region values, which are applied far enough from the wall to have little influence.

The velocity shift Δu^+ is finally obtained as the difference between the solution for a smooth wall and

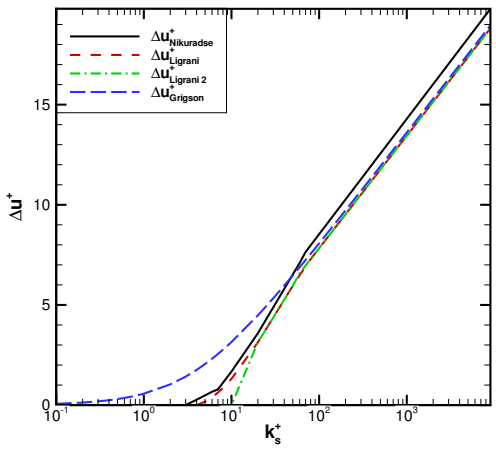

Figure 2: Comparison of correlations for the shift of the logarithmic region proposed by Nikuradse, Ligrani and Moffat for sand grain (Ligrani) and hemispheres (Ligrani 2) and Grigson for Colebrook's experiments

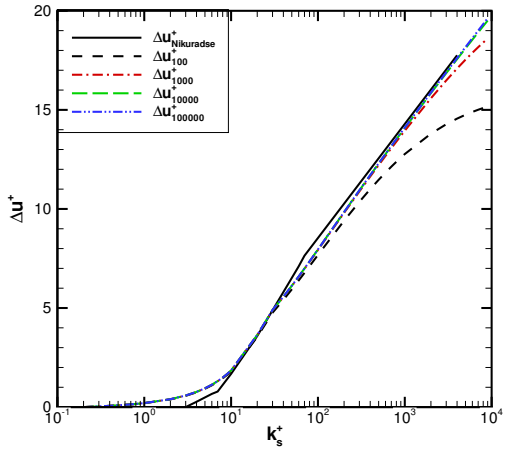

Figure 3: Velocity shift of the logarithmic region – Wilcox' (1988) $k-\omega$ model and roughness correction

the one for a given roughness correction. It has to be checked that the logarithmic region is just shifted, not distorted, so that this difference will be plotted for various altitudes in the following figures.

Wilcox' correction

Together with his $k-\omega$ model, Wilcox (1988) proposed to correct the model to account for wall roughness. Only the specific dissipation is modified as

$$\omega_w = \frac{u_\tau^2 S_R}{\nu} \qquad S_R = \begin{cases} \left(\frac{50}{k_s^+}\right)^2 & \text{if } k_s^+ \leq 25 \\ \frac{100}{k_s^+} & \text{if } 25 \leq k_s^+ \leq 2000 \end{cases}$$

i.e. a finite specific dissipation is now imposed at the wall, as a decrease in dissipation will lead to an enhanced turbulence compared to a smooth wall, and thus higher momentum transfer towards the wall and higher friction.

The evolution of the velocity shift Δu^+ versus the reduced sand grain roughness height k_s^+ is plotted in figure 3. The Nikuradse correlation is given as a reference and the various curves correspond to the velocity shift computed at different altitudes (e.g. Δu^+_{1000} is evaluated at $y^+ \approx 1000$). As long as the reduced sand grain roughness height k_s^+ is smaller than the altitude at which the velocity shift is computed, all curves are superimposed, which confirms that the logarithmic region is just shifted. Moreover, the shift is in nice agreement with Nikuradse's correlation, except for small roughness ($k_s^+ < 10$) where the correction gives larger values. It has to be mentioned that Wilcox (2008) proposed a correction to force the velocity shift to remain negligible for $k_s^+ < 5$ but this model gives an unrealistic behaviour in the transition regime.

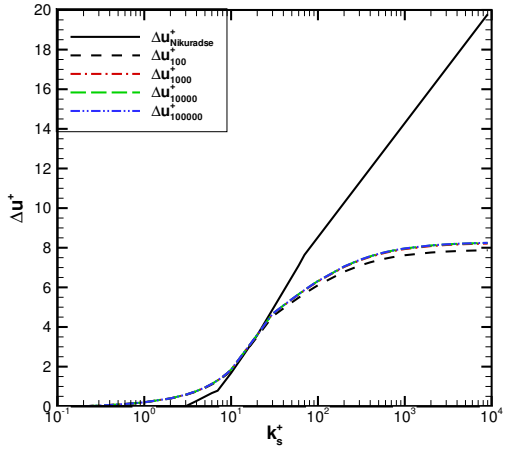

Figure 4: Velocity shift of the logarithmic region – Menter's $k-\omega$ SST model and Wilcox' (1988) roughness correction

When Wilcox' (1988) correction is applied with the SST model, the velocity shift is underestimated for reduced sand grain roughness heights k_s^+ larger than 30, as shown in figure 4. This can easily be explained considering the rôle of the SST limiter. The eddy viscosity reduction, i.e. the ratio between the eddy viscosity with and without the SST limiter, or $\min\left(1; \frac{a_1\omega}{\Omega F_2}\right)$ is plotted in figure 5 for various reduced equivalent sand grain heights k_s^+. This figure shows that, even on a smooth wall, the SST limiter is slightly active but, as the roughness increases, the turbulence level increases so that the limiter is more and more active and significantly reduces the eddy viscosity, leading to poor predictions.

Hellsten and Laine correction

To cope with this problem, Hellsten and Laine

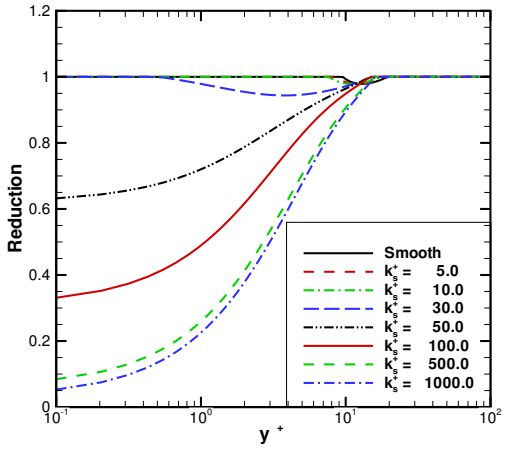

Figure 5: Reduction of the eddy viscosity by the SST limiter – Menter's $k-\omega$ SST model and Wilcox' (1988) roughness correction

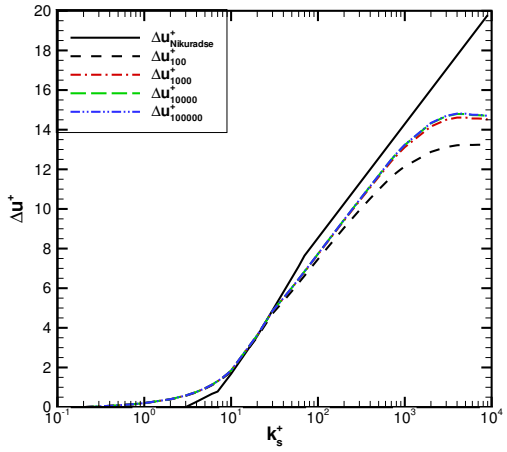

Figure 6: Velocity shift of the logarithmic region – Menter's $k-\omega$ SST model and Hellsten and Laine roughness correction

(1998) proposed to inhibit the SST limiter in the wall region by introducing the function

$$F_3 = 1 - \tanh\left[\left(\frac{150\nu}{\omega y^2}\right)^4\right]$$

in the limiter. A simple evaluation of this function using logarithmic region solutions shows that it shuts down the limiter close to the wall but let it active in the logarithmic region. Figure 6 shows that this correction works nicely as a correct behaviour is retrieved, except for very large roughness ($k_s^+ > 1000$) for which the analysis of the eddy viscosity reduction shows that reduction starts to occur for reduced wall distances y^+ larger than 60 and can become significant.

It can also be noticed that, opposite to the previous figures, the predicted velocity shift is now smaller than Nikuradse's correlation, closer to Ligrani and Moffat or Grigson's correlations. This is due to the change of the diffusion coefficient σ_k in the SST model.

Knopp et al. correction

Knopp et al. claimed some numerical issues with the Hellsten and Laine correction and developed a new correction, inspired by the strategy proposed by Aupoix and Spalart (2003). Their correction imposes finite wall values for both the turbulent kinetic and specific dissipation rate as

$$k_w = \frac{u_\tau^2}{\sqrt{\beta^*}}\min\left(1, \frac{k_s^+}{90.}\right) \qquad \omega_w = \frac{u_\tau}{\sqrt{\beta^*}\kappa d_0}$$

$$d_0 = 0.03 k_s \min\left[1, \left(\frac{k_s^+}{30.}\right)^{2/3}\right] \min\left[1, \left(\frac{k_s^+}{45.}\right)^{1/4}\right] \min\left[1, \left(\frac{k_s^+}{60.}\right)^{1/4}\right]$$

Their correction is calibrated w.r.t. the Ligrani and Moffat correlation so that this one is also plotted in figure 7 which shows that the correction works well in the fully rough regime, for $k_s^+ > 100$ but strongly underestimates the velocity shift in the transition regime.

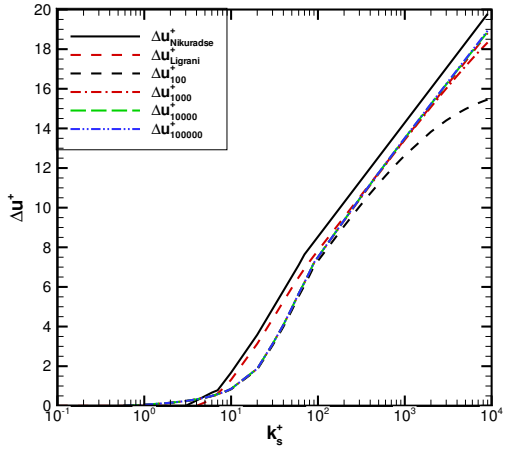

Figure 7: Velocity shift of the logarithmic region – Menter's $k-\omega$ SST model and Knopp et al. roughness correction

3 Derivation of new roughness corrections

Wilcox' correction fails when coupled with the SST model, as is well known, while Hellsten and Laine correction works but gives slightly lower friction level, and gets into trouble for very large roughness. Knopp et al. correction is poor in the transition regime. Moreover, it was wished to be able to evaluate different behaviours in the transition regime. All these arguments motivated the development of new corrections. The strategy is the one already applied for the Spalart and Allmaras model (Aupoix and Spalart (2003) and

the Smith's $k-l$ model (Aupoix (2007)) and is detailed in the second reference.

The basic idea is, for a given reduced roughness height k_s^+, to impose as wall conditions for the flow over a rough surface the solution given by the model over a smooth surface at a distance from the wall where the velocity is equal to the desired velocity shift $\Delta u^+(k_s^+)$ so that, as the wall region is a constant total shear stress region, the whole solution is just shifted by Δu^+. Moreover, the analysis of the correction behaviour for very small roughness heights as well as in the fully rough regime is analytical and provides guidelines to derive expressions for the the wall conditions.

Two such corrections were derived for the Menter's $k-\omega$ SST model, one based upon the Nikuradse's correlation and one upon the Grigson's representation of Colebrook's results. It could be argued that it is somehow inconsistent to use a correction with a von Kármán constant κ of 0.40 with a model which is tuned to predict a value of 0.41. The use of these two correlations roughly gives an envelop of the velocity shift.

For the Nikuradse's correlation the so-obtained wall conditions read

$$
\begin{aligned}
k_w^+ &= \max\left(0; k_0^+\right) \\
k_0^+ &= \frac{1}{\sqrt{\beta^*}} \tanh\left[\left(\frac{\ln\frac{k_s^+}{30}}{\ln 8} + 0,5\left[1-\tanh\frac{k_s^+}{100}\right]\right)\right. \\
&\quad \left.\tanh\left(\frac{k_s^+}{75}\right)\right] \\
\omega_w^+ &= \frac{400000}{k_s^{+4}}\left(\tanh\frac{10000}{3k_s^{+3}}\right)^{-1} \\
&\quad + \frac{70}{k_s^+}\left[1-\exp\left(-\frac{k_s^+}{300}\right)\right]
\end{aligned}
$$

while for the Colebrook data, they read

$$
\begin{aligned}
k_w^+ &= \max\left(0; k_0^+\right) \\
k_0^+ &= \frac{1}{\sqrt{\beta^*}} \tanh\left[\left(\frac{\ln\frac{k_s^+}{30}}{\ln 10} + 1 - \tanh\frac{k_s^+}{125}\right)\right. \\
&\quad \left.\tanh\frac{k_s^+}{125}\right] \\
\omega_w^+ &= \frac{300}{k_s^{+2}}\left(\tanh\frac{15}{4k_s^+}\right)^{-1} + \\
&\quad \frac{191}{k_s^+}\left(1-\exp\left(-\frac{k_s^+}{250}\right)\right)
\end{aligned}
$$

It was checked that the above representations are accurate within a few percent to the numerically determined values.

The wall distance should also be shifted, although this shift is quite small, about three percent of the equivalent sand grain roughness height. In the SST model, the wall distance only appears in the functions F_1 and F_2. It can be analytically proved that neglecting the shift in the wall distance leads to an activation of the SST limiter but this limiter activation is marginal and it was checked that the model, without shift of the wall distance, predicts velocity shifts in nice agreement with the corresponding correlation, as shown in figure 8.

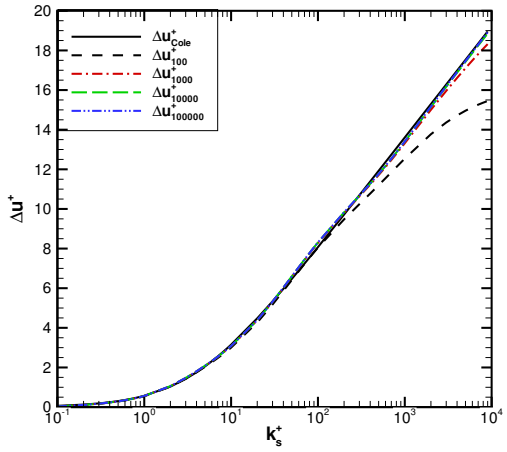

Figure 8: Velocity shift of the logarithmic region – Menter's $k-\omega$ SST model and present roughness correction for the Colebrook's data representation by Grigson

4 Roughness correction assessment

Introduction

The roughness correction assessment was performed using the ONERA two-dimensional boundary layer code CLICET which has a self-adaptive grid and several grid levels so that grid convergence was easily checked. Computations were performed with some classical turbulence models with wall roughness corrections and all the above presented $k-\omega$ roughness corrections applied to both the BSL and SST models. There is not enough room to present all this amount of results so that mainly results with the SST model for the most relevant validation cases will be discussed here.

All figures are similar, with a lower solid line corresponding to the SST model predictions over a smooth wall, and the model predictions with Wilcox' correction (solid line, labeled SST Menter), Knopp et al. correction (dash line, labeled SST Menter Knopp), present corrections respectively based upon Nikuradse (long dash line, labeled SST Menter Niku) and Colebrook (dash dot line, labeled SST Menter Cole) based correlations, and at last Hellsten and Laine correction (dash dot dot line, labeled SST Menter Hell).

MSU experiments

L/D	2	4
Grabow and White	1.1	0.29
Waigh and Kind	1.58	0.38
Flack and Schultz	3.5	4.8
Experiments	1.16 ± 0.26	0.45 ± 0.02

Table 1: Equivalent sand grain roughness heights (in millimeters) for the MSU experiments for the two ratio spacing over diameter L/D

A large number of experimental investigations of flows over rough surfaces were conducted at the Mississippi State University, considering pipe flows and boundary layer flows. Roughness elements were hemispheres or cones, regularly distributed. Three different test cases for flows over hemispheres are reported here, to cover a wide range of reduced sand grain roughness k_s^+. Boundary layer experiments by Hosni et al. (1991,1993), dealing with hemispheres of 0.635mm height, with a spacing of two or four diameters, are considered here.

For a so simple roughness geometry, it seems a priori easy to determine the equivalent sand grain roughness. Three correlations were used; the first one by Grabow and White (1975) is widely used, the second one by Waigh and Kind (1998) is dedicated to such regularly distributed roughness elements, and the last one is the recent one by Flack and Schultz (2010). The equivalent sand grain roughness heights predicted by these correlations are compared to the values determined from experiments in table 1. It appears that the correlations poorly predict the experimental sand grain roughness height, which will be used in the computations.

For these experiments, they are some problems in the determination of the initial conditions so that the results are not plotted versus the distance but versus the Reynolds number based upon the momentum thickness $R\theta$ to avoid some bias.

For a spacing of four diameters and a velocity of 12ms^{-1}, the reduced equivalent sand grain height k_s^+ is about 15, i.e. the beginning of the transition regime. Roughness increases the wall friction by about 40%, as can be seen in figure 9. Wilcox's and Hellsten and Laine corrections are superimposed, which confirms that there is no detrimental effect of the SST limiter for so small a roughness. Their results are close to the one obtained with the Nikuradse based correction, but far below the experiments. Knopp et al. correction underestimates even more the wall friction in this regime, as already shown with the simplified analysis. Only the Colebrook based correction, which gives stronger roughness effects in the transition regime, is in fair agreement with the experiments.

For a spacing of four diameters and a velocity of 58ms^{-1}, the reduced equivalent sand grain height k_s^+ is about 65, i.e. the end of the transition regime.

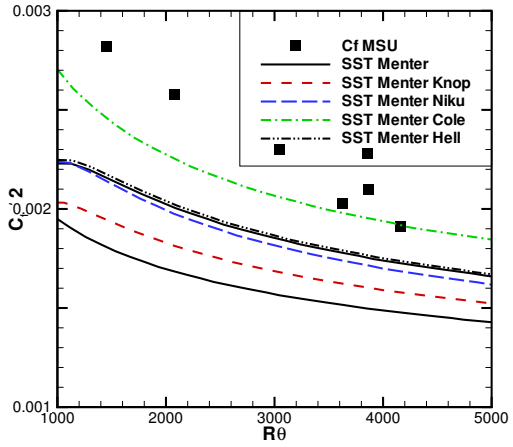

Figure 9: MSU experiments – External velocity 12 ms^{-1} – Spacing over diameter ratio of 4

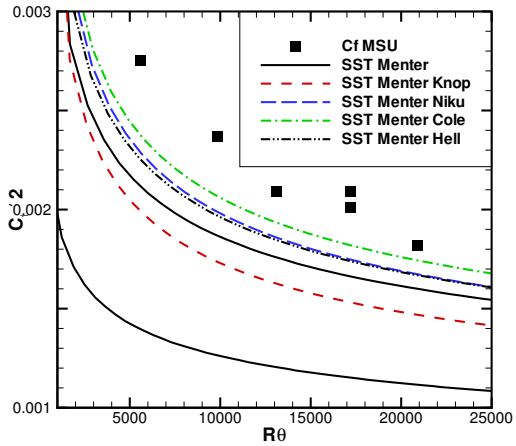

Figure 10: MSU experiments – External velocity 58 ms^{-1} – Spacing over diameter ratio of 4

Roughness increases the wall friction by about 80%, as can be seen in figure 10. The situation has changed as now the Hellsten and Laine correction predicts higher friction levels than Wilcox' correction, due to the interference of Wilcox' correction with the SST limiter. Hellsten and Laine correction is again in close agreement with the Nikuradse based correction, a little below the Colebrook based correction, the difference being much smaller than previously. Knopp et al. correction still underestimates the friction. Again, this correction ranking is in agreement with the simplified analysis.

For a spacing of two diameters and a velocity of 58ms^{-1}, the reduced equivalent sand grain height k_s^+ is about 300, i.e. definitely in the fully rough regime. Roughness increases the wall friction by a factor close to three, as can be seen in figure 11. The failure of the Wilcox' correction is now obvious, while Knopp et

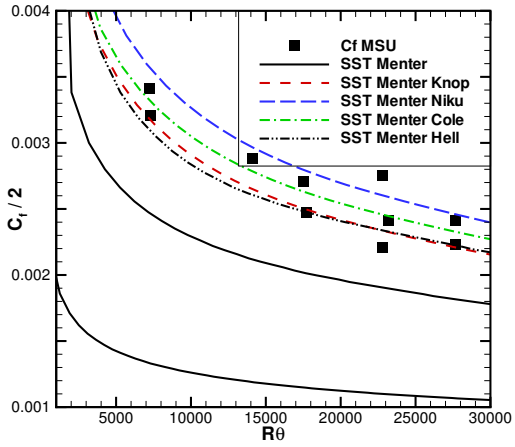

Figure 11: MSU experiments – External velocity 58 ms^{-1} – Spacing over diameter ratio of 2

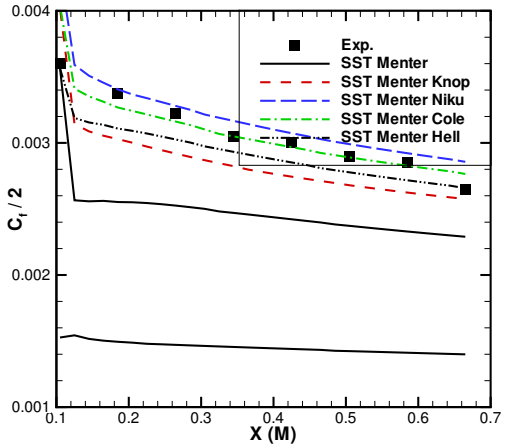

Figure 13: Blanchard's experiments – Roughness of 0.425mm – Zero pressure gradient

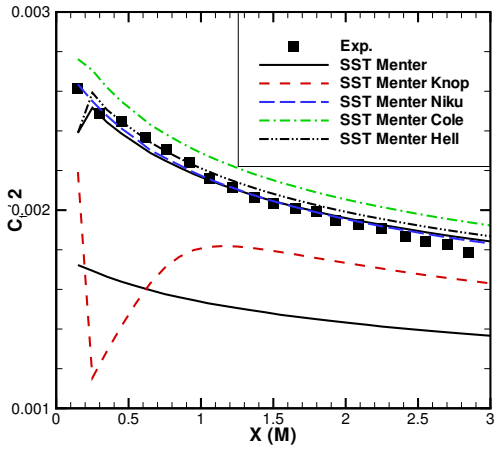

Figure 12: Acharya et al. experiments – SRS1 surface

al. correction, which is well calibrated for this regime, is in close agreement with Hellsten and Laine correction, the Colebrook based correction being close. The Nikuradse based correction now yields higher friction levels. Again, this correction ranking is in agreement with the simplified analysis.

Acharya et al. experiments

At the Brown Bowery research center, Acharya et al. (1986) performed experiments on surfaces designed to simulate aged turbine blade surfaces, therefore named SRS (Simulated Roughness Surface). SRS2 gives an equivalent sand grain roughness about 70 and results very similar to the second MSU case above. Only SRS1 is discussed here.

The equivalent sand grain roughness was determined from the modelling of the surface by Tarada (1987) and the Grabow and White correlation, and was taken as 1.064mm. The equivalent sand grain height is about 25, i.e. the transition regime, which leads to a friction increase about 40%. The SST limiter starts playing a rôle so that the Wilcox' correction is slightly below the Hellsten and Laine one, as shown in figure 12. Nikuradse based correction is nearly superimposed with Wilcox' one, while higher friction levels are predicted by the Colebrook based correction. At last, Knopp et al. correction fails again, as expected in the transition regime.

Blanchard's experiments

While previous experiments dealt with zero pressure gradient flows, Blanchard (1977), at ONERA, performed experiments on sand paper of various sizes and wire mesh, without and with pressure gradient. Only the sand paper with a sand grain of 0.425mm is considered here, results for the sand grain of 0.58mm, which correspond to a reduced equivalent sand grain height k_s^+ about 200 are similar to those obtained for the rougher MSU case.

For this sand paper, Blanchard proposes a model to represent the roughness element shape based upon pyramids, but this model leads to a very different equivalent sand grain roughness height from the one he determined. The experimentally determined sand grain roughness height, of twice the roughness element height, is used here.

Results for the flow without pressure gradient are plotted in figure 13. The reduced sand grain roughness height k_s^+ is about 150, i.e. in the fully rough regime, which leads to a doubling of the wall friction compared to the smooth case and complements the domain covered by the MSU experiments. Here again, the difference between Wilcox' correlation and the Hellsten and Laine one evidences the detrimental effect of the SST limiter. Knopp et al. correlation still leads to too small friction levels. For the fully rough regime, the Nikuradse based correction yields higher friction lev-

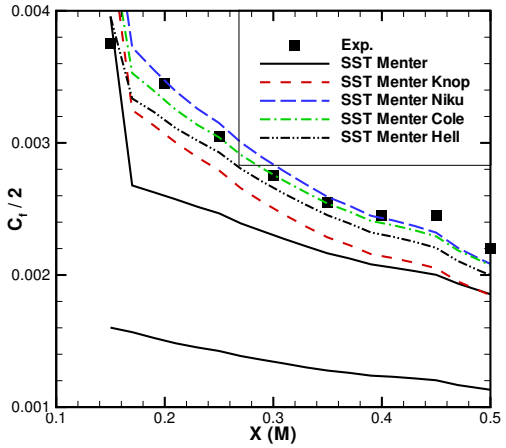

Figure 14: Blanchard's experiments – Roughness of 0.425mm – Adverse pressure gradient

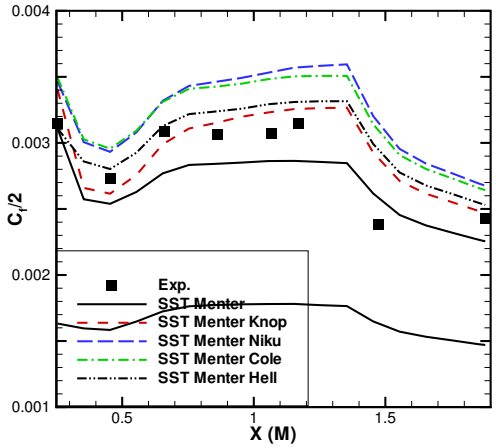

Figure 15: Coleman et al. experiments – Most densely packed hemispheres of 1.27mm diameter – Case 3: Equilibrium flow under negative pressure gradient

els than the Colebrook based one and the Hellsten and Laine one.

To the author's knowledge, Blanchard provides the only experiment of boundary layer flow over a rough surface in presence of an adverse pressure gradient, which is enough documented to be used as a validation test case in a boundary layer framework. The adverse pressure gradient is not strong enough to lead to separation but a significant reduction of the friction coefficient is however observed. One can notice from figure 14 that the friction reduction is much stronger on the rough surface ($\approx$ 40%) than on the smooth surface ($\approx$ 25%). This can be partly linked to the loss of flow momentum in the wall region due to the presence of the roughness. Although roughness effects are very grossly modelled in the equivalent sand grain approach, the decrease of the friction coefficient is well reproduced. As the reduced sand grain roughness height is similar to the previous case, the correction ranking is the same.

Coleman et al. experiments

At Stanford University, the team of Moffat and Kays investigated boundary layer flows over spheres of 1.27mm diameter, packed in the most dense arrangement. Among all the works performed on this surface, the ones by Coleman et al. (1976,1977) were selected as they deal with accelerated boundary layer flows.

Case 3 of an equilibrium flow and case 5 of a non-equilibrium flows were investigated, only results from case 3 are presented here, the conclusions being similar.

This surface raises new problems as there is some flow in the lower part of the spheres. Therefore, the equivalent sand grain roughness height used by Coleman et al., and borrowed from Schlichting (1936) who investigated the same surface, of 1.25 times the sphere diameter, is used.

Figure 15 shows that an equilibrium regime, characterized by a constant skin friction coefficient, is achieved over a significant part of the experiment. As the reduced sand grain roughness height k_s^+ is about 100, the flow is in the fully rough regime and the friction level is increased by about 80%. One can notice that the same friction increase was obtained in the MSU experiments, at 58 ms^{-1} and a spacing of four, for a lower value of k_s^+ of about 65. This evidences the coupled action of flow acceleration and roughness, acceleration bringing more momentum to the fluid in the wall region and leading to an enhanced roughness effect, as well as the ability of the roughness corrections to reproduce it. Although it is an accelerated flow, the SST limiter plays a rôle and deteriorates the prediction, Knopp et al. correction works well in the fully rough regime and yields predictions very close to Hellsten and Laine correction while Coleman based correction gives a slightly higher level and Nikuradse based correction a little higher. Even with accelerated flows, the correction ranking remains consistent with the simplified analysis, once based upon the reduced equivalent sand grain height k_s^+.

At last, it can be mentioned that Spalart and Allmaras or mixing length models, with roughness corrections, similarly overestimate the wall friction compared to experiments.

Further comments

Computations performed with the BSL model (not shown) lead to the same ranking between the corrections as with the SST model all over the roughness regimes, Wilcox' correction taking here the place of the Hellsten and Laine one in the absence of SST limiter. Slightly higher friction levels are systematically obtained using the BSL model, compared to the SST

model predictions, this being due to the change of the diffusion coefficient σ_k. These comparisons also show that results very close to those obtained with Wilcox' correction are obtained with the Nikuradse based correction for the transition regime and with the Colebrook based one in the fully rough regime, in agreement with the simplified analysis.

So, these corrections can be used with any $k-\omega$ model but their use with $k-\omega$ models using near-wall corrections to reproduce the near-wall peak of the turbulent kinetic energy is not recommended as the wall region solution is altered.

5 Conclusion

All the above test cases confirmed the failure of the Wilcox' correction coupled with the SST model over most of the transition regime and all the fully rough regime. As no very large roughness was addressed, Hellsten and Laine correction did not fail but gives slightly lower friction levels, while Knopp et al. correction underestimates wall friction in the transition regime and the beginning of the fully rough regime. Both proposed corrections provide friction levels slightly higher than Hellsten and Laine, this being partly due to the fact that the Hellsten and Laine correction is based upon Wilcox' correction which was tuned for a different value of the diffusion coefficient σ_k. The Colebrook based correction predicts higher friction levels in the transition regime, as it was designed too. The ability of models to capture coupled effects of wall roughness and pressure gradients was also evidenced, as well for positive as negative presurre gradients. All these results are fully consistent with the simplified analysis and validate it, even in presence of pressure gradients.

It is thus finally recommended to use the Colebrook based correction, because on the one hand it is designed to provide higher levels in the transition regime and, on the other hand, it is much consistent with the value of the von Kármán constant given by the SST model and thus gives better predictions in the fully rough regime. The Nikuradse based correction can be used to complement it and have a first estimate of bounds in the transition regime. The corrections can also be blended, taking the larger (resp. smaller) k_w^+ together with the smaller (resp. larger) ω_w^+ to obtain a larger envelope.

Another lesson learned from these test cases is the difficulty to determine the equivalent sand grain roughness for a given surface, and thus to determine the "best" correction, as little change in the sand grain roughness can alter the conclusions.

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